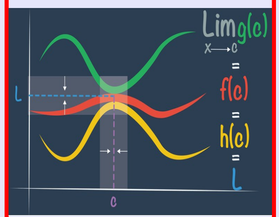
Calculus I

Lecture 17



Feb 19-8:47 AM

Class QZ 14

1) Evaluate $\int (5x^4 - 6\sqrt[5]{x}\,)\,dx$ $\quad \sqrt[5]{x} \to x^{1/5}$

$= \frac{\cancel{5}x^5}{\cancel{5}} - 6\,\frac{x^{\frac{1}{5}+1}}{\frac{1}{5}+1} + C = x^5 - \frac{6x^{\frac{6}{5}}}{\frac{6}{5}} + C$

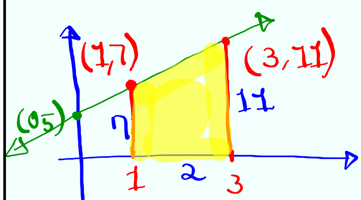
$= x^5 - \frac{\cancel{6}\cdot 5}{\cancel{6}}\, x\sqrt[5]{x} + C$

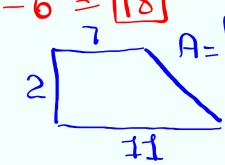
$= \boxed{x^5 - 5x\sqrt[5]{x} + C}$

2) Find $\int_1^3 (2x+5)\,dx$

$= (x^2+5x)\Big|_1^3 = [3^2+5(3)] - [1^2+5(1)]$

$= 24 - 6 = \boxed{18}$





$A = \frac{h(B+b)}{2}$

$= \frac{\cancel{2}(11+7)}{\cancel{2}}$

$= \boxed{18}$

Feb 3-7:47 AM

Integration Rules

1) $\int k f(x)\,dx = k\int f(x)\,dx$

2) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

3) $\int k\,dx = kx + C$

4) $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C \quad , n \neq -1$

5) $\int \sin x\,dx = -\cos x + C$

6) $\int \cos x\,dx = \sin x + C$

7) $\int \sec^2 x\,dx = \tan x + C$

8) $\int \csc^2 x\,dx = -\cot x + C$

9) $\int \sec x \tan x\,dx = \sec x + C$

10) $\int \csc x \cot x\,dx = -\csc x + C$

Feb 2-8:38 AM

Evaluate

1) $\int (x^2 - 3\sqrt[3]{x} + 2)\,dx = \dfrac{x^3}{3} - 3\cdot\dfrac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + 2x + C$ $\sqrt[3]{x} \to x^{1/3}$

$= \dfrac{1}{3}x^3 - \dfrac{3x^{4/3}}{4/3} + 2x + C$

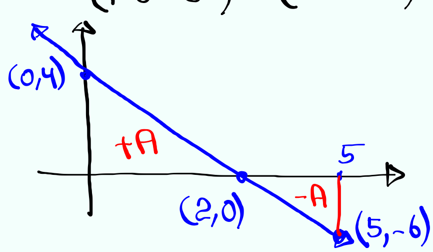
$= \dfrac{1}{3}x^3 - 3\cdot\dfrac{3}{4}x^{4/3} + 2x + C$

$= \boxed{\dfrac{1}{3}x^3 - \dfrac{9}{4}x\sqrt[3]{x} + 2x + C}$

2) $\int_2^5 (4-2x)\,dx$

$= (4x - x^2)\Big|_2^5$

$= (4\cdot 5 - 5^2) - (4\cdot 2 - 2^2) = -5 - 4 = \boxed{-9}$



Feb 3-8:26 AM

3) $\int \frac{x^3 - 2\sqrt{x}}{x}\,dx = \int \left[\frac{x^3}{x} - \frac{2\sqrt{x}}{x}\right]dx$ $\qquad \frac{1}{x^n} = x^{-n}$

$= \int \left(x^2 - \frac{2}{\sqrt{x}}\right)dx = \int \left[x^2 - 2x^{-1/2}\right]dx = \frac{x^3}{3} - \frac{2x^{1/2}}{1/2} + C$

$= \boxed{\frac{1}{3}x^3 - 4\sqrt{x} + C}$

$(A+B)^2$

4) $\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$ $\qquad \frac{\sqrt{x}}{x}\cdot\frac{\sqrt{x}}{\sqrt{x}} = \frac{x}{x\sqrt{x}} = \frac{1}{\sqrt{x}}$

$A^2 + 2AB + B^2$ $\qquad \frac{\sqrt{x}}{x} = \frac{x^{1/2}}{x} = x^{\frac{1}{2}-1} = x^{-1/2}$

$= \int_1^2 \left(x^2 + 2\cdot x \cdot \frac{1}{x} + \frac{1}{x^2}\right)dx$

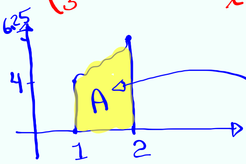
$= \int_1^2 \left(x^2 + 2 + x^{-2}\right)dx = \left(\frac{x^3}{3} + 2x + \frac{x^{-1}}{-1}\right)\Big|_1^2$

$= \left(\frac{1}{3}x^3 + 2x - \frac{1}{x}\right)\Big|_1^2 = \left(\frac{1}{3}\cdot 2^3 + 2\cdot 2 - \frac{1}{2}\right) - \left(\frac{1}{3} + 2 - 1\right)$

$= \frac{8}{3} + 4 - \frac{1}{2} - \frac{1}{3} - 1$

$= \frac{7}{3} + 4 - \frac{3}{2} = \frac{14 + 24 - 9}{6}$

$= \boxed{\frac{29}{6}}$



Feb 3-8:37 AM

5) $\int (x - \csc x \cot x)\,dx = \frac{x^2}{2} - (-\csc x) + C$

$= \boxed{\frac{1}{2}x^2 + \csc x + C}$

6) $\int_1^8 \frac{x-1}{\sqrt[3]{x^2}}\,dx = \int_1^8 \left[\frac{x}{\sqrt[3]{x^2}} - \frac{1}{\sqrt[3]{x^2}}\right]dx$

$= \int_1^8 \left[\frac{x^1}{x^{2/3}} - \frac{1}{x^{2/3}}\right]dx = \int_1^8 \left(x^{1/3} - x^{-2/3}\right)dx$

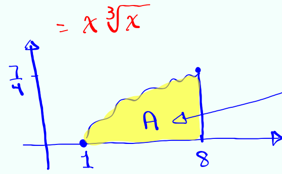
$= \left(\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}\right)\Big|_1^8 = \left(\frac{x^{4/3}}{4/3} - \frac{x^{1/3}}{1/3}\right)\Big|_1^8$

$= \left(\frac{3}{4}x\sqrt[3]{x} - 3\sqrt[3]{x}\right)\Big|_1^8$

$x^{4/3} = \sqrt[3]{x^4}$ $\qquad = \left(\frac{3}{4}\cdot 8 \cdot \sqrt[3]{8} - 3\sqrt[3]{8}\right) - \left(\frac{3}{4} - 3\right)$

$= \sqrt[3]{x^3}\sqrt[3]{x}$ $\qquad = 12 - 6 - \frac{3}{4} + 3 = 9 - \frac{3}{4}$

$= x\sqrt[3]{x}$ $\qquad = \boxed{\frac{33}{4}}$



Feb 3-8:54 AM

Evaluate $\int_{-1}^{2} (x - 2|x|)\,dx$

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

$|x| = -x$ $\quad$ $|x| = x$

$-1 \quad 0 \quad 2$

$$= \int_{-1}^{0} (x - 2(-x))\,dx + \int_{0}^{2} (x - 2x)\,dx$$

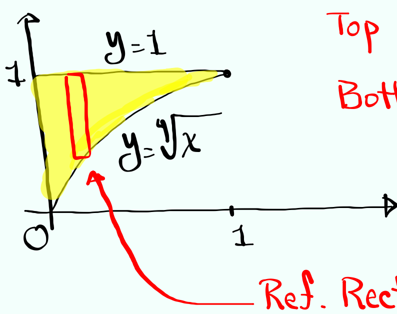
$$= \int_{-1}^{0} 3x\,dx + \int_{0}^{2} -x\,dx = \frac{3x^2}{2}\Big|_{-1}^{0} - \frac{x^2}{2}\Big|_{0}^{2}$$

$$= \frac{3}{2}\left(0^2 - (-1)^2\right) - \frac{1}{2}\left(2^2 - 0^2\right)$$

$$= \frac{3}{2}\cdot(-1) - 2 = \frac{-3}{2} - 2 = \boxed{-\frac{7}{2}}$$

Feb 3-9:10 AM

Find the shaded area below



Top $\quad y = 1$

Bottom $\quad y = \sqrt[4]{x}$ $\quad x^{1/4}$

$$A = \int_{0}^{1} \left[1 - \sqrt[4]{x}\right] dx$$

$$= \left(x - \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1}\right)\Bigg|_{0}^{1} = \left(x - \frac{x^{5/4}}{5/4}\right)\Bigg|_{0}^{1}$$

$$= \left(1 - \frac{1}{5/4}\right) - \left(0 - \frac{0}{5/4}\right)$$

$$= 1 - \frac{4}{5} - 0 = \boxed{\frac{1}{5}}$$

Feb 3-9:17 AM

use Subs. method to evaluate

$$\int \frac{1}{(5x-1)^4}\,dx \quad \text{using } u = 5x-1.$$

$du = 5\,dx$

$\frac{du}{5} = dx$

$$= \int \frac{1}{u^4}\cdot\frac{du}{5} = \frac{1}{5}\int u^{-4}\,du$$

$$= \frac{1}{5}\cdot\frac{u^{-3}}{-3} + C = \frac{-1}{15}(5x-1)^{-3} + C$$

$$= \frac{-1}{15(5x-1)^3} + C$$

Feb 3-9:24 AM

$$\int x\sqrt{x^2+1}\,dx \quad , \quad u = x^2+1$$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

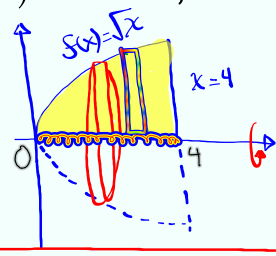
$$= \int \sqrt{u}\,\frac{du}{2}$$

$$= \frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}\,u\sqrt{u} + C$$

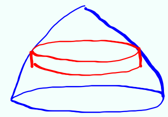
$$\boxed{= \frac{1}{3}(x^2+1)\sqrt{x^2+1} + C}$$

Feb 3-9:32 AM

Rotate the region bounded by $f(x)=\sqrt{x}$, x-axis, and $x=4$ around x-axis.



It will generate a volume



when region is totally attached to A.O.R. and Ref. Rect. ⊥ to A.O.R.

Method $\boxed{\text{Disk Method}}$

$$V=\int_0^4 \pi\left[f(x)\right]^2dx=\int_0^4\pi(\sqrt{x})^2dx$$

$$=\pi\int_0^4 x\,dx$$

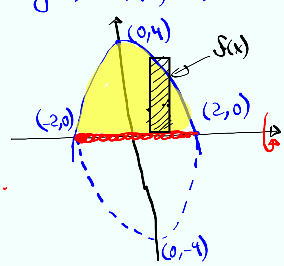
$$=\pi\cdot\frac{x^2}{2}\Big|_0^4$$

$$\pi\left(\frac{4^2}{2}-\frac{0^2}{2}\right)$$

$$=\boxed{8\pi}$$

Feb 3-10:04 AM

Consider the region enclosed by $f(x)=4-x^2$ and x-axis. Rotate by x-axis, find the volume.



1) Region is totally attached to A.O.R.

2) Ref. Rect. ⊥ A.O.R.

Disk Method

$$V=\int_{-2}^{2}\pi\left[4-x^2\right]^2dx=2\int_0^2\pi(4-x^2)^2dx$$

$$=2\pi\int_0^2(16-8x^2+x^4)dx$$

$$=2\pi\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]\Big|_0^2$$

$$=2\pi\left[\left(16\cdot2-\frac{8\cdot2^3}{3}+\frac{2^5}{5}\right)-(0)\right]$$

$$=2\pi\left[32-\frac{64}{3}+\frac{32}{5}\right]=2\pi\cdot\frac{256}{15}=\boxed{\frac{512\pi}{15}}$$

Feb 3-10:13 AM

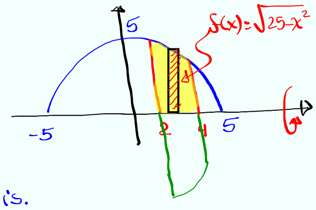
Consider the region bounded by

$y = \sqrt{25 - x^2}$, $y = 0$, $x = 2$, and $x = 4$.

1) Draw & shade

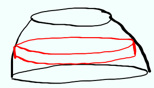
$y^2 = 25 - x^2$

$x^2 + y^2 = 25$



2) Rotate about x-axis.

Region totally attached to A.O.R.



Ref. Rect. $\perp$ A.O.R.

Disk Method

$$V = \int_2^4 \pi \left[\sqrt{25-x^2}\right]^2 dx = \pi \int_2^4 (25 - x^2)\,dx$$

height of Rectangle

$$= \pi \left[25x - \frac{x^3}{3} \right] \Bigg|_2^4 = \boxed{\frac{94\pi}{3}}$$

$$= \pi \left[\left(25 \cdot 4 - \frac{4^3}{3}\right) - \left(25 \cdot 2 - \frac{2^3}{3}\right) \right]$$

$$= \pi \left[100 - \frac{64}{3} - 50 + \frac{8}{3} \right] = \pi \left[50 - \frac{56}{3} \right] = \pi \cdot \frac{94}{3}$$

Feb 3-10:24 AM

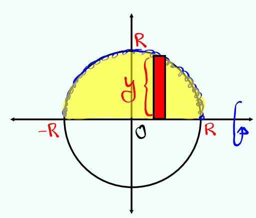
Consider the region bounded by the top-half of a Circle with radius R and centered at the origion and x-axis.

1) Draw & clearly label.

$x^2 + y^2 = R^2$

$y^2 = R^2 - x^2$

$y = \sqrt{R^2 - x^2}$



Rotate by x-axis, find its Volume.

Disk Method

$$V = \int_{-R}^{R} \pi \left[\sqrt{R^2 - x^2}\right]^2 dx = \pi \cdot 2 \int_0^R \left[R^2 - x^2\right] dx$$

height of Ref. Rect.

$$= 2\pi \left[R^2 x - \frac{x^3}{3} \right] \Bigg|_0^R = 2\pi \left[R^2 R - \frac{R^3}{3} - 0 \right]$$

$$= 2\pi \left[R^3 - \frac{R^3}{3} \right]$$

$$= 2\pi \cdot R^3 \left[1 - \frac{1}{3} \right]$$

$$= 2\pi R^3 \cdot \frac{2}{3} = \boxed{\frac{4\pi R^3}{3}}$$

Feb 3-10:35 AM

Draw the region bounded by

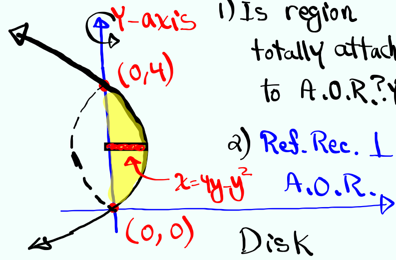
$x = 4y - y^2$ and $x = 0$.

Parabola opens left

$x = 0$

$4y - y^2 = 0$

$y(4 - y) = 0$



1) Is region totally attached to A.O.R.? Yes

2) Ref. Rec. $\perp$ A.O.R.

Disk

Rotate about Y-axis

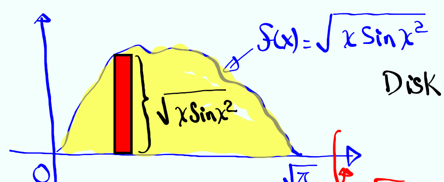
$$V = \int_0^4 \pi \underbrace{[4y - y^2]}_{\text{Length of Ref. Rect.}}{}^2 \, dy = \pi \int_0^4 [16y^2 - 8y^3 + y^4] \, dy$$

$$= \pi \left[\frac{16y^3}{3} - \frac{8y^4}{4} + \frac{y^5}{5} \right] \Big|_0^4$$

$$= \pi \left[\frac{16 \cdot 4^3}{3} - 2(4)^4 + \frac{4^5}{5} - 0 \right] = \boxed{\frac{512\pi}{15}}$$

Feb 3-10:47 AM

Rotate the region given below by the x-axis, find its volume.



Disk ✓

$$V = \int_0^{\sqrt{\pi}} \pi \left[\sqrt{x \sin x^2} \right]^2 dx = \pi \int_0^{\sqrt{\pi}} x \sin x^2 \, dx$$

$u = x^2$

$du = 2x \, dx$

$\frac{du}{2} = x \, dx$

$x = 0 \quad u = 0^2 = 0$

$x = \sqrt{\pi} \quad u = (\sqrt{\pi})^2 = \pi$

$$= \pi \int_0^{\pi} \sin u \, \frac{du}{2}$$

$$= \frac{\pi}{2} \cdot -\cos u \Big|_0^{\pi}$$

$$= \frac{-\pi}{2} \left[\overset{-1}{\cancel{\cos \pi}} - \overset{1}{\cancel{\cos 0}} \right]$$

$$= \frac{-\pi}{2} \cdot (-2) = \boxed{\pi}$$

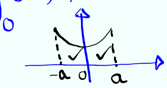
Feb 3-11:02 AM

Special integration:

1) $\int_a^a f(x)\,dx = \boxed{0}$

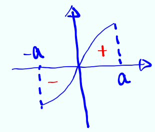
2) If $f(x)$ is even,

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$



3) If $f(x)$ is odd,

$$\int_{-a}^{a} f(x)\,dx = 0$$



Ex. $\int_2^2 \tan^3(x^2-1000x)\,dx = 0$

Ex: Suppose $\int_0^5 f(x)\,dx = 10$ and $f(x)$ is even

$$\int_{-5}^{5} f(x)\,dx = 2\int_0^5 f(x)\,dx = 2(10) = 20$$

Ex: $\int_{-10}^{10} x(x^2+2)^{20}\,dx$

Feb 3-11:16 AM

Ex: $\int_{-10}^{10} x(x^2+2)^{20}\,dx$

Method I

$u = x^2+2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x=-10 \quad u=102$

$x=10 \quad u=102$

$$= \int_{102}^{102} u^{20}\,\frac{du}{2} = \boxed{0}$$

Method II:

$f(x) = x(x^2+2)^{20}$

$f(-x) = -x((-x)^2+2)^{20} = -x(x^2+2)^{20} = -f(x)$

$f(x)$ is odd

$$\int_{-10}^{10} \text{odd function}\,dx = \boxed{0}$$

Feb 3-11:25 AM

Evaluate $\int x^2 \sqrt{x+2}\, dx$

Hint: $u = \sqrt{x+2}$

$u^2 = x+2$

$2u\,du = dx$

$u^2 - 2 = x$

$= \int (u^2-2)^2\, u \; 2u\,du$

$= 2\int u^2(u^4 - 4u^2 + 4)\,du$

$= 2\int [u^6 - 4u^4 + 4u^2]\,du$

$= 2\left[\frac{u^7}{7} - \frac{4u^5}{5} + \frac{4u^3}{3}\right] + C$

$= 2\left[\frac{(\sqrt{x+2})^7}{7} - \frac{4(\sqrt{x+2})^5}{5} + \frac{4(\sqrt{x+2})^3}{3}\right] + C$

Feb 3-11:30 AM

Evaluate

$\int x^3 \sqrt{x^2+1}\, dx$

Hint 1: $u = x^2+1$

Hint 2: $x^3 = x^2 \cdot x$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$u - 1 = x^2$

$= \int x^2 \cdot x \sqrt{x^2+1}\, dx$

$= \int (u-1)\sqrt{u}\,\frac{du}{2}$

$= \frac{1}{2}\int (u^{3/2} - u^{1/2})\,du = \frac{1}{2}\left[\frac{u^{5/2}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$

Feb 3-11:39 AM

Open notes

Class QZ 15

1) Evaluate $\int_0^{\sqrt{\pi}} 2x \cos x^2 \, dx$

$u = x^2 \quad du = 2x\,dx$

$x = 0 \rightarrow u = 0$

$x = \sqrt{\pi} \rightarrow u = \pi$

$= \int_0^{\pi} \cos u \, du = \sin u \Big|_0^{\pi} = \sin \pi - \sin 0$

$= \boxed{0}$

2) Evaluate $\int (\tan x + 1)^3 \sec^2 x \, dx$

$u = \tan x + 1$

$du = \sec^2 x \, dx$

$= \int u^3 \, du = \frac{u^4}{4} + C$

$= \boxed{\frac{1}{4}(\tan x + 1)^4 + C}$

Feb 3-11:49 AM